Formal Methods for Computer System Design and Analysis

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Ist. di Scienza e Tecnologie dell'Informazione "A. Faedo"
Formal Methods && Tools Lab

SEFM 2010

These slides are available at: http://www.sefm2010.isti.cnr.it/school/docs/introduction_and_motivations_latella.pdf

Outline

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1. Background

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2. Formal Methods for Software Engineering;

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5. Success Stories;
6. Extensions;

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- quite incomplete !!

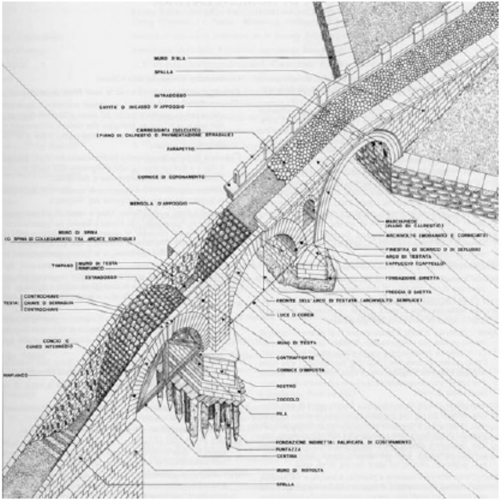
Background - Engineering

"All engineering disciplines make progress by employing mathematically based notations and methods."

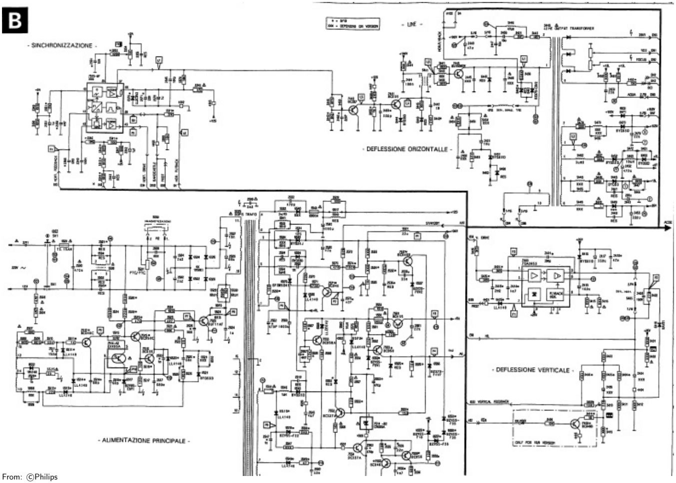
[C. Jones 2000]

Background - Engineering - Notations

Graphical



©M. Mazzoleni, L. Jurina "PONTI IN MURATURA: DIFETTI E PATOLOGIE", CIAS 2006, Bolzano



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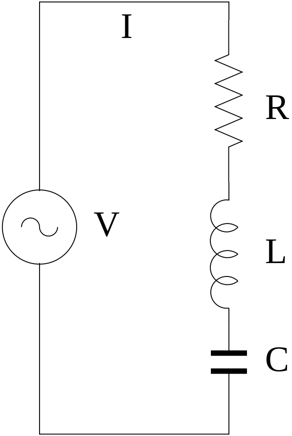

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- **Ways for composing them**

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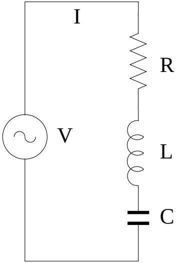
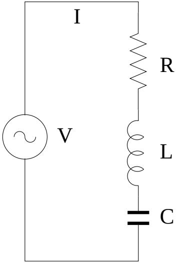
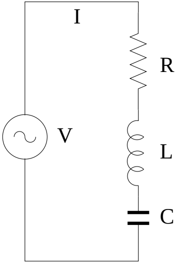




circuit definition:

CIRCUIT RLC $(x_v, x_r, x_l, x_c) \triangleq$
CONNECT $(x_v$, SERIES (RES(x_r), IND(x_l), CAP(x_c)))



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circuit use (instantiation):

RLC(**V,R,L,C**)

Mathematically based Notations

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- Rigorous (Formal) Syntax

- Rigorous (Formal) Semantics

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  - ~~$\sqrt{75 + :} \quad \frac{2}{0}$~~
- Rigorous (Formal) Semantics
  - Set Theory, Relations and Functions
  - Continuous Mathematics
    - *Metric Spaces*
    - *Differential Calculus and Function Analysis*
    - *Linear Algebra*
    - *Differential Equations*
  - Mathematical Logic
  - *... and much more!*

Background - Engineering - Notations

Semantics: abstract definitions

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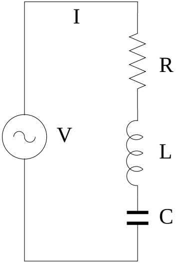

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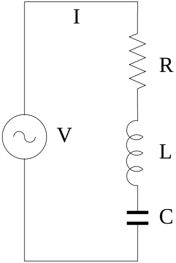

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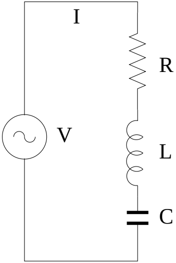

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$$v(t) = R \cdot i(t) + L \cdot \frac{d\, i(t)}{d\, t} + \frac{1}{C} \cdot \int_{-\infty}^{t} i(x)\, d\, x$$

Semantics: methods for formal manipulation

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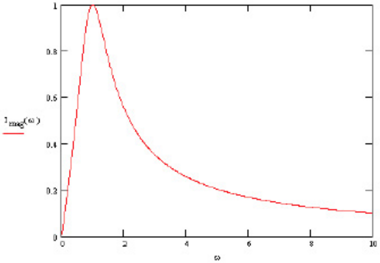
$$\frac{d^2 q(t)}{dt} + \frac{R}{L} \cdot \frac{d\,q(t)}{d\,t} + \frac{1}{LC} \cdot q(t) = \frac{1}{L} \cdot v(t)$$

Background - Engineering - Notations

Semantics: identification and study of characteristic features



$I_{max}(\omega)$: current magnitude as a function of frequency ω

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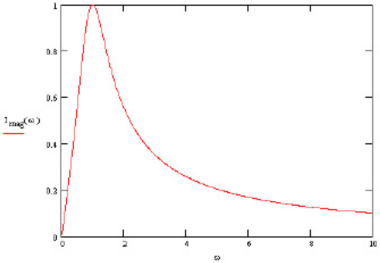

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the value ω_0 s.t. I_{max} has a peak in ω_0

Background - Engineering - Notations

Computer support: mechanization of formal manipulation

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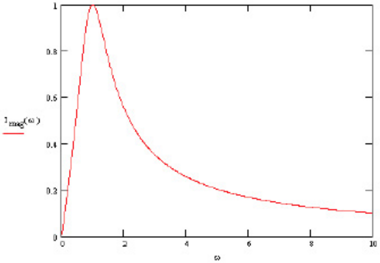

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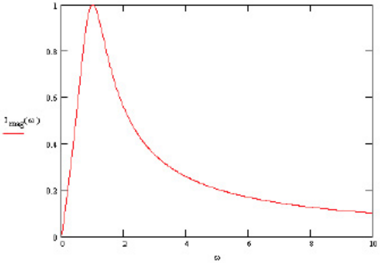

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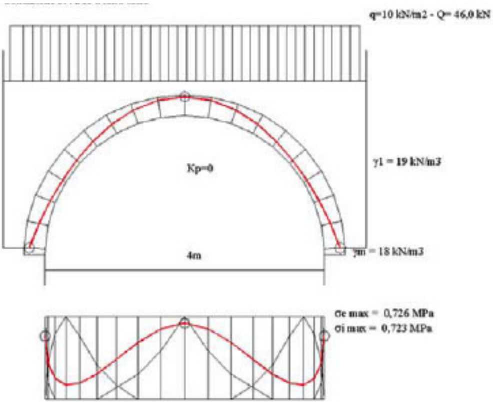
Technical Specification

- Resonance frequency: 10.000 Hz
- ...

Sample technical specification: Audio Power Amplifier

Power output	25 W rms per channel
Load impedence	8 ohms
Total distorsion	$< 0.08\%$
Frequency response	$10 \div 50.000$ Hz ($+0.5$ dB, -2 dB)
Power requirements	
Power requirements	220 V (50 Hz)
Max power comsumption	160 W
Dimensions	430 mm W
	132 mm H
	247 mm D
Weight	4.5 Kg


q=10 kN/m2 - Q= 46,0 kN
Kp=0
γ1 = 19 kN/m3
4m
γm = 18 kN/m3
σe max = 0,726 MPa
σi max = 0,723 MPa

Sample technical specification: The bridge

Max load .... ??? t
Max wind speed .... ??? m/s
Oscillation Freq. .... ??? Hz
. . .

Background - Engineering - Technical Specifications

In general terms, *Technical Specifications*:

- are characteristic features of the system
- describe desirable requirements on the system
- help reasoning about the system
- should be met by the system

In general terms, *Technical Specifications*:

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Civil, Naval, Nuclear, Electrical, Electronic (. . .) Engineers use notations for

- $\surd$ **technical specifications (requirements specifications)** as well as
- $\surd$ **design specifications/models**

which

- are strongly based on mathematics (and physics),
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- allow for the formal manipulation of their objects
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Engineers are supposed to be aware of underlying theories but they are not required to completely master them.

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What about (Critical) Software Engineers?

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What is the mathematical basis of SE?

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Research on 'formal methods' follows this model and attempts to identify and develop mathematical approaches that can contribute to the task of creating computer systems"

[C. Jones 2000]

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Attempt to provide the (software) engineer with "concepts and techniques as thinking tools, which are clean, adequate, and convenient, to support him (or her) in describing, reasoning about, and constructing complex software and hardware systems"

[W. Thomas 2000]

Formal Methods

$$\text{Applying} \left\{ \frac{\text{Logic in}}{\text{Theoretical}} \right\} \text{Computer Science}$$

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For Supporting System Engineering

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For Supporting System Engineering

Emphasis on

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- Automatic, often push-botton, Software Tool Support

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For Supporting System Engineering

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rather than

Formal Methods

Applying $\left\{ \frac{\text{Logic in}}{\text{Theoretical}} \right\}$ Computer Science

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rather than

- classical issues like completeness.

Formal Methods - OUR FOCUS

- Here we focus on concurrent systems
  - System: composed of (very) many components
  - Component: performs (very) simple tasks (often sequential)
  - Interaction: complex; difficult to understand; non-deterministic; subtle (race conditions, synchronization issues, dead-/live-locks, etc.)

However notice that

sound mathematical theories for non-concurrent, sequential (functional, imperative) programs exist

- the bulk of Computation Theory (Gödel, Turing, Church, etc)
- *formal semantics, e.g.*
  - Operational Semantics
    (based on abstract machines)
  - Denotational semantics
    (based on lattices, complete partial orders, fixpoint theory)
- *formal analysis, e.g.*
  - Hoare Logic
  - Cousot Abstract Interpretation

A
Labelled Transition Systems
Process Algebraic
Approach to
System Modelling
(design specification)

State-Transition Structures

Description of behaviour of a system

- A set of *states* which the system can be at
- A set of *transitions* which describe *how* a system can move from *which* state to *which* one

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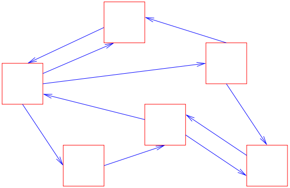
The specific set of values of variables and execution points (PC) of the SW components of a distributed system at a given point in time (i.e. the system **state vector**)	Execution of a command (e.g. variable assignment)
Being *free* or *in use* of a computing resource in a system	Granting (or refusing) a request of use

State -Transition Structures

- **Graphical notation**



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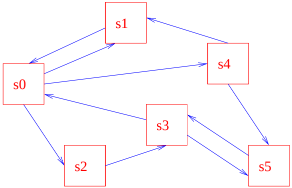

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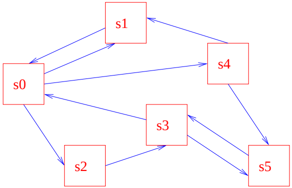

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State -Transition Structures

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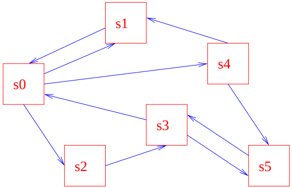

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- S is the set of states

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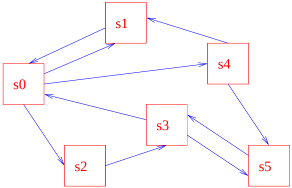

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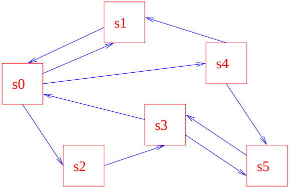

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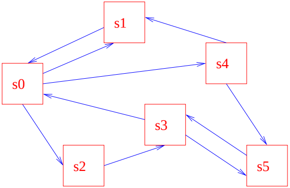
- $\rightarrow \subseteq S \times S$ is the transition relation

- **Graphical notation**



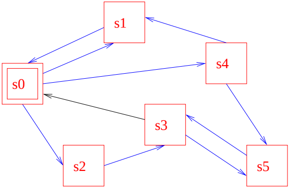
- **Mathematical definition**

A STS is a tuple $(S, \rightarrow)$ where:

- S is the set of states ($\{s_0, s_1, s_2, s_3, s_4, s_5\}$ in the example above)
- $\rightarrow \subseteq S \times S$ is the transition relation
  ($\rightarrow = \{(s_0, s_1), (s_1, s_0), (s_0, s_2), \ldots, (s_4, s_5)\}$ in the example above)

State -Transition Structures

- **Graphical notation**



- **Mathematical definition**

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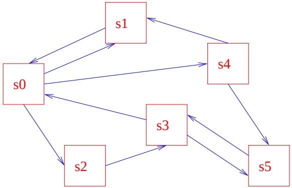
State -Transition Structures

- **Graphical notation**



- **Mathematical definition**

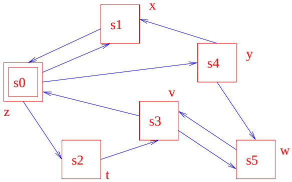
A STS is a tuple $(S, \rightarrow)$ where:

- S is the set of states ($\{s_0, s_1, s_2, s_3, s_4, s_5\}$ in the example above)
- $\rightarrow \subseteq S \times S$ is the transition relation ($\rightarrow = \{(s_0, s_1), (s_1, s_0), (s_0, s_2), \ldots, (s_4, s_5)\}$ in the example above)
- $s_0 \in S$ is the initial state

We write $s \rightarrow s'$ whenever $(s, s') \in \rightarrow$

State Labelled-Transition Structures

- **Graphical notation**

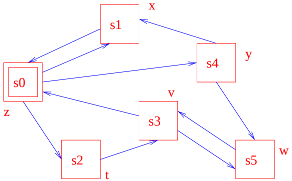


- **Mathematical definition**

A tuple $(S, A_s, L, \rightarrow, s_0)$ where:

- S is the set of states
- A_s is a set of state labels (in the example $\{x, y, v, w, t, z\}$)
- $L : S \longrightarrow A_s$ is a state-labelling function
    - *e.g.* $L(s)$ *is the* state vector *at* s,
    - *or* $L(s)$=*ok iff a given component is up and running in* s
    - *etc.*
- $\rightarrow \subseteq S \times S$ is the transition relation
- $s_0 \in S$ is the initial state

- **Graphical notation**



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A tuple $(S, A_s, L, \rightarrow, s_0)$ where:

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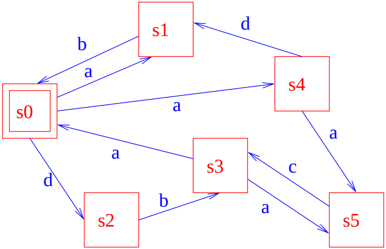
  - *etc.*
- $\rightarrow \subseteq S \times S$ is the transition relation
- $s_0 \in S$ is the initial state

- *Kripke Structure*: $L(s)$ is a set of *atomic propositions* holding in s

State-Transition Labelled Structures

- **Graphical notation**

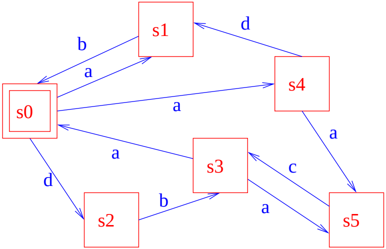


- **Mathematical definition**

  A tuple $(S, A_t, \rightarrow, s_0)$ where:
  - S is the set of states
  - A_t is a set of transition labels (actions) (in the example $\{a, b, c, d\}$)

    a may denote an interaction (e.g. synchronous communication) or a local operation (e.g. assignment)
  - $\rightarrow \subseteq S \times A_t \times S$ is the transition relation
  - $s_0 \in S$ is the initial state

- **Graphical notation**



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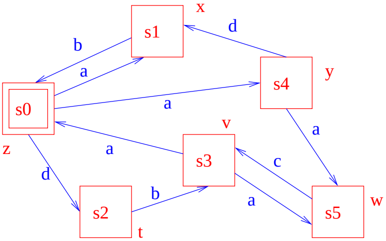
- *Labelled Transition Systems (LTS)*

State and Transition Labelled Structures

- **Graphical notation**



- **Mathematical definition**

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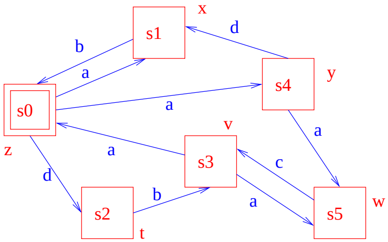
- **Graphical notation**



- **Mathematical definition**

  A tuple $(S, A_s, L, A_t, \rightarrow, s_0)$ where:
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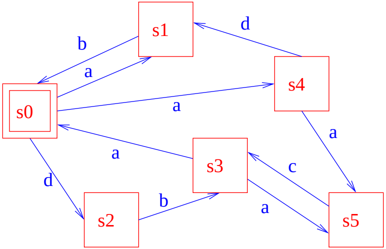
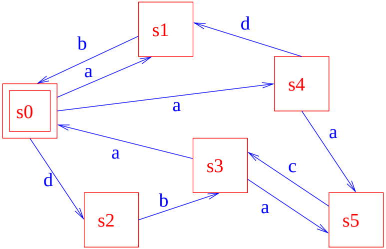
  - $L : S \longrightarrow A_s$ is a state-labelling function
  - $\rightarrow \subseteq S \times A_t \times S$
  - $s_0 \in S$ is the initial state
- *Doubly Labelled Transition Systems / Bi-Labelled Transition Systems ([De Nicola, Vaandeager]/ [Gnesi et al.])*

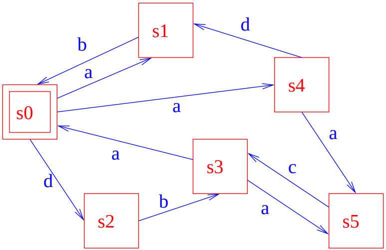
Example of Textual definition of a LTS



Example of Textual definition of a LTS



$$
\begin{aligned}
s0 &\triangleq a.s1 + d.s2 + a.s4 \\
s1 &\triangleq b.s0 \\
s2 &\triangleq b.s3 \\
s3 &\triangleq a.s0 + a.s5 \\
s4 &\triangleq d.s1 + a.s5 \\
s5 &\triangleq c.s3
\end{aligned}
$$

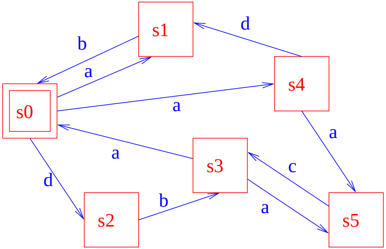


$$\begin{array}{lll} s0 & \triangleq & a.s1 + d.s2 + a.s4 \\ s1 & \triangleq & b.s0 \\ s2 & \triangleq & b.s3 \\ s3 & \triangleq & a.s0 + a.s5 \\ s4 & \triangleq & d.s1 + a.s5 \\ s5 & \triangleq & c.s3 \end{array}$$

Formal Syntax definition of *process* states

$$\begin{array}{llll} S & ::= & \mathbf{nil} & (\textit{no action}) \\ & \mid & \alpha.S & (\textit{action prefix}) \\ & \mid & S + S & (\textit{choice}) \\ & \mid & \alpha.X & (\textit{constant } X) \end{array}$$

Example of Textual definition of a LTS



$$s0 \triangleq a.s1 + d.s2 + a.s4$$
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Formal Syntax definition of ***process*** **states**

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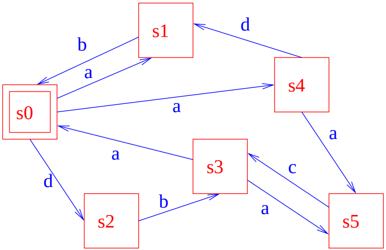
\end{array}
$$

with actions $\alpha \in A_t$

Example of Textual definition of a LTS



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Formal Syntax definition of *process* states

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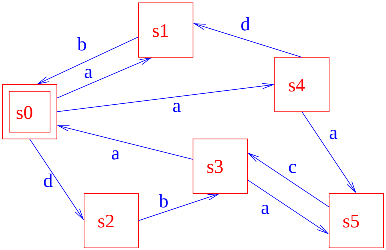
\end{array}
$$

with actions $\alpha \in A_t$

and constants X defined via equations $X \triangleq S$

Example of Textual definition of a LTS



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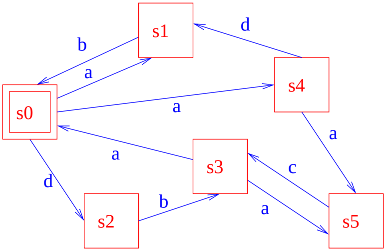
- **Basic components**
- **Ways for composing them**

Example of Textual definition of a LTS



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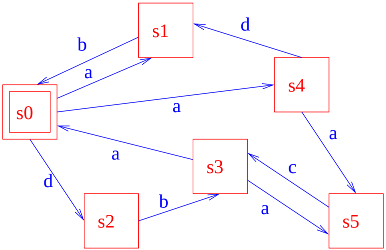
\end{aligned}
$$

- **Basic components**
  e.g. Resistors, Inductances, Capacitors
- **Ways for composing them**

Example of Textual definition of a LTS



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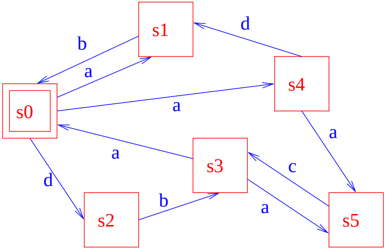
- **Basic components**
  e.g. **nil**, *Actions*

- **Ways for composing them**

Example of Textual definition of a LTS



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$$

- **Basic components**
  e.g. **nil**, *Actions*
- **Ways for composing them**
  e.g. action prefix operator $(\_.\_)$, *choice operator* $(\_ + \_)$

From algebraic terms to LTS via Formal Semantics.

$$s0 \triangleq a.s1 + d.s2 + a.s4$$
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From algebraic terms to LTS via Formal Semantics.

Formal Syntax definition (Grammar)

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From algebraic terms to LTS via Formal Semantics.

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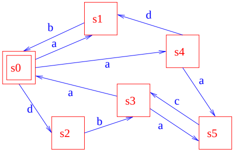

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(Grammar)

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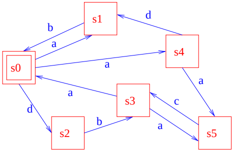
\end{aligned}
$$

Mathematical Objects
(LTS)



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Formal Syntax definition
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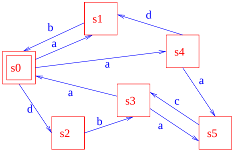
Mathematical Objects
(LTS)



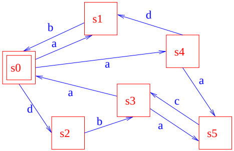
From algebraic terms to LTS via Formal Semantics.

Formal Syntax definition (Grammar)

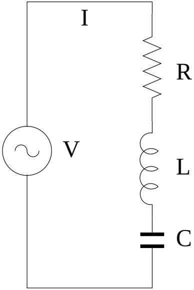
$$\begin{aligned} s0 &\triangleq a.s1 + d.s2 + a.s4 \\ s1 &\triangleq b.s0 \\ s2 &\triangleq b.s3 \\ s3 &\triangleq a.s0 + a.s5 \\ s4 &\triangleq d.s1 + a.s5 \\ s5 &\triangleq c.s3 \end{aligned}$$

Formal Semantics definition (Logic deduction system)

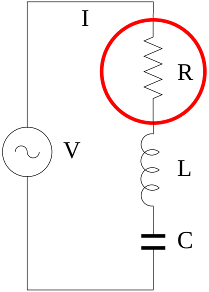
Mathematical Objects (LTS)



Back to the RLC circuit



Focus on R



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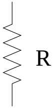
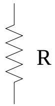
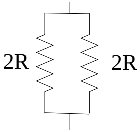
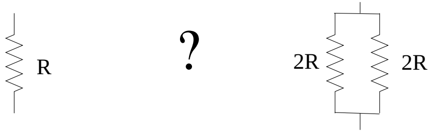


RES(**R**) and PARALLEL (RES(**2R**),RES(**2R**))

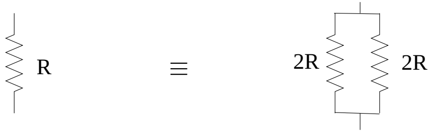




RES(**R**) and PARALLEL (RES(**2R**),RES(**2R**))



RES(**R**) ≡ PARALLEL (RES(**2R**),RES(**2R**))

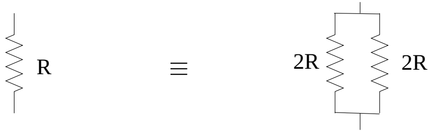


RES(**R**) $\equiv$ PARALLEL (RES(**2R**),RES(**2R**))

It can be proved:

$$Resistance(PARALLEL(RES(R_1), RES(R_2))) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

RES(**R**) $\equiv$ PARALLEL (RES(**2R**),RES(**2R**))

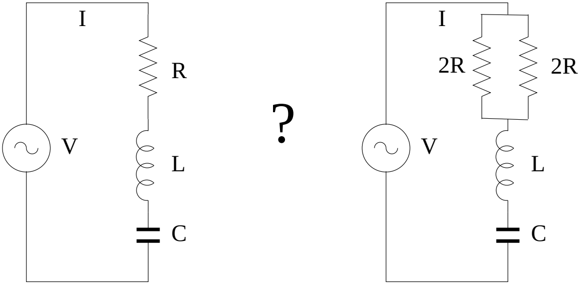


It can be proved:

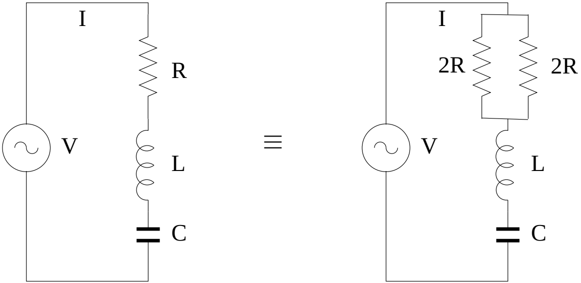
$$Resistance(PARALLEL(RES(R_1), RES(R_2))) = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$Resistance(PARALLEL_{j=1}^{k}(RES(R_j))) = \frac{1}{\sum_{j=1}^{k}\frac{1}{R_j}}$$

Replacing equivalent components ...

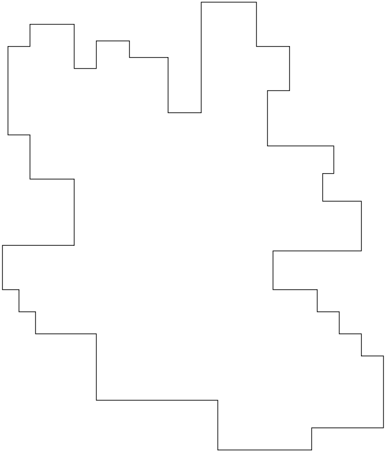


... brings to equivalent circuits



Congruence

Congruence



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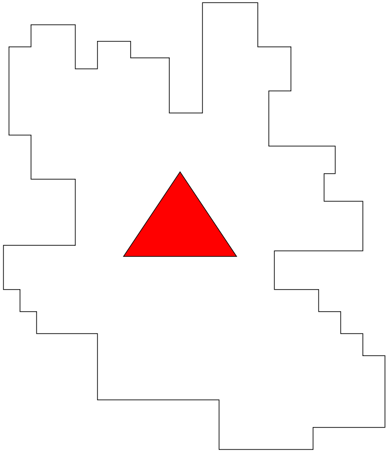

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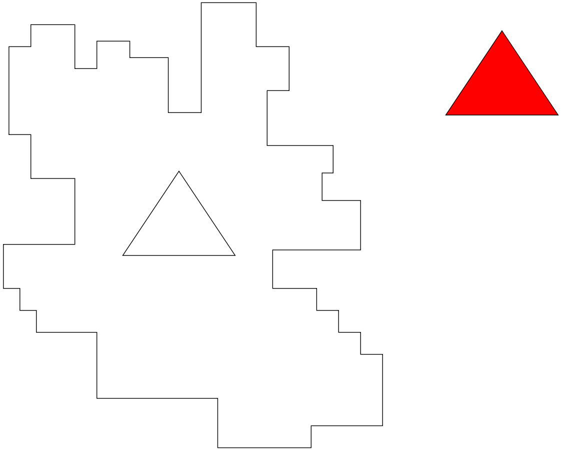

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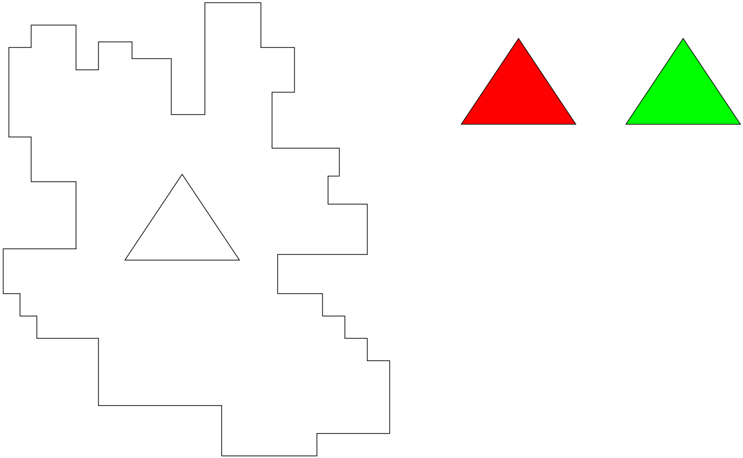

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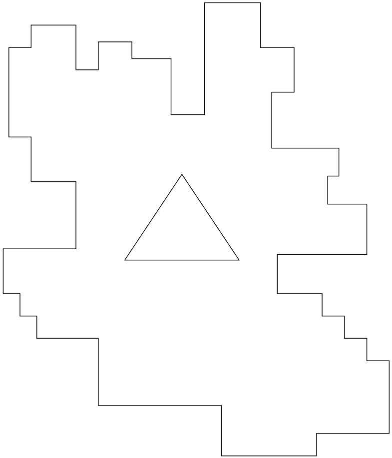
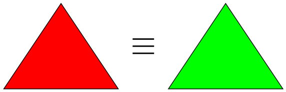
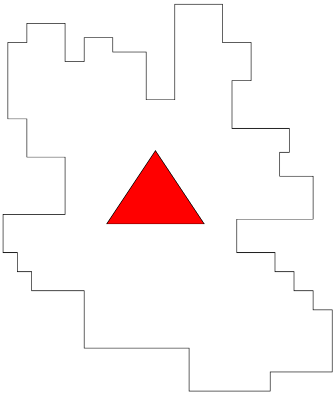
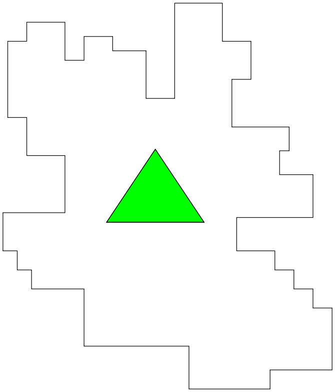




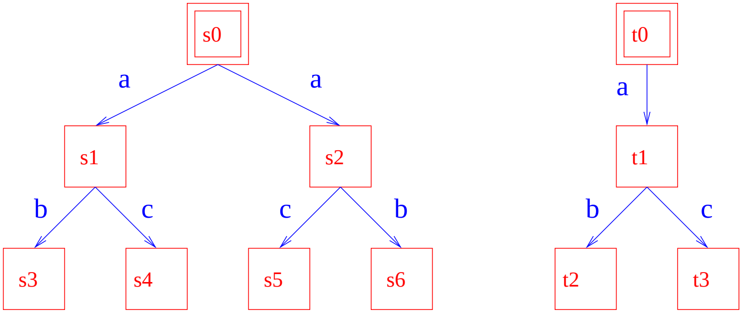
Congruence



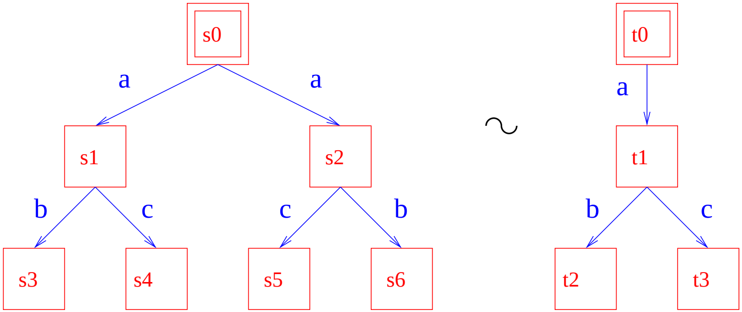
$\equiv$



LTS behaviour equivalence



LTS behaviour equivalence



(Strong) Bisimulation Equivalence

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Two states s and t are *Bisimulation Equivalent* ($s \sim t$) iff there exists *bisimulation relation* $\mathcal{B}$ s.t. $s \, \mathcal{B} \, t$

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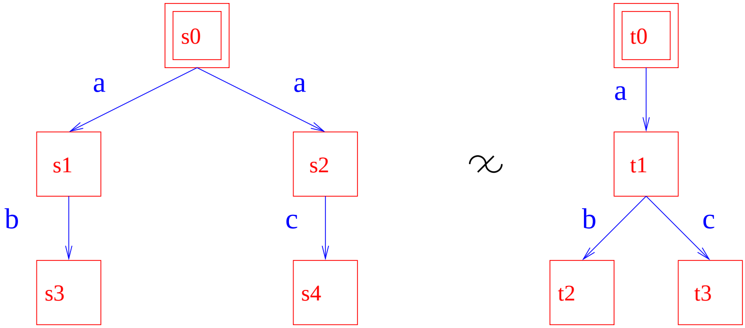
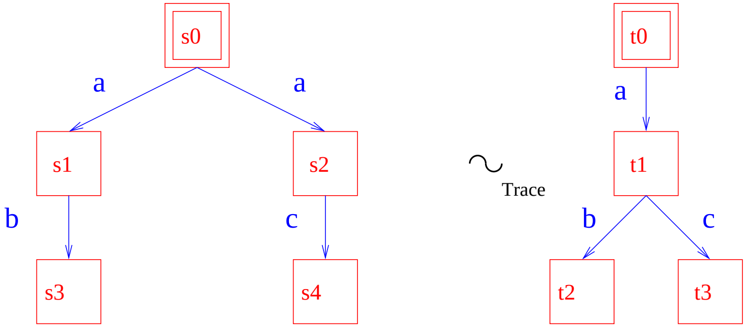
We usually refer to the *initial* states of two systems.

LTS behaviour equivalence



LTS behaviour equivalence



Algebraic Laws for Bisimulation Equivalence

Algebraic Laws for Bisimulation Equivalence

For all terms S, S_1, S_2

$$
\begin{aligned}
S + \mathbf{nil} &\sim S \\
S + S &\sim S \\
S_1 + S_2 &\sim S_2 + S_1 \\
S + (S_1 + S_2) &\sim (S + S_1) + S_2
\end{aligned}
$$

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For all terms S, S_1, S_2

$$\begin{array}{rcl} S + \mathbf{nil} & \sim & S \\ S + S & \sim & S \\ S_1 + S_2 & \sim & S_2 + S_1 \\ S + (S_1 + S_2) & \sim & (S + S_1) + S_2 \end{array}$$

Bisimulation Equivalence is actually a *congruence*:
If $S_1 \sim S_2$ then, for all S and α

$$\begin{array}{rcl} \alpha.S_1 & \sim & \alpha.S_2 \\ S + S_1 & \sim & S + S_2 \end{array}$$

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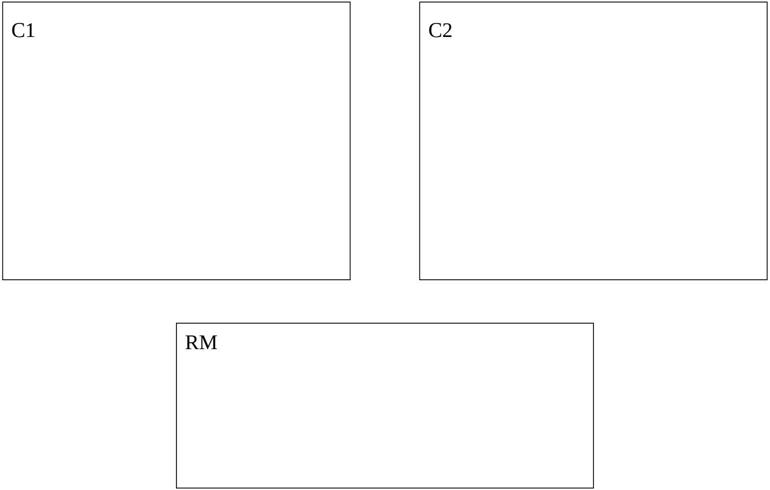
\end{array}
$$

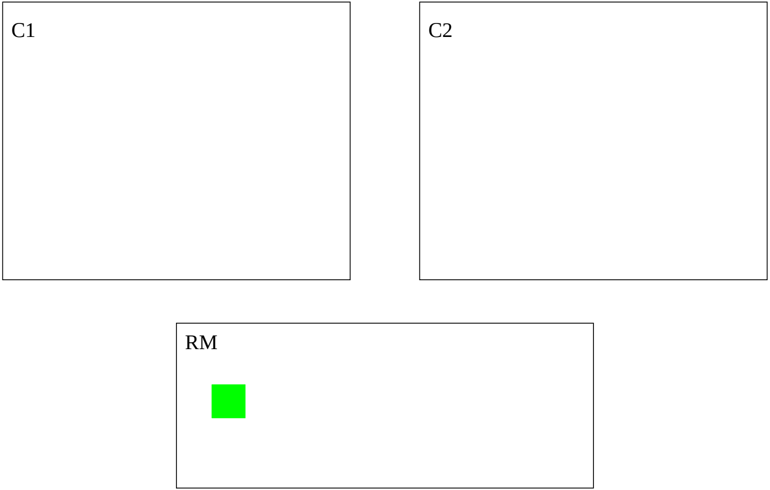
Expressions can be *reduced/simplified*!!

Parallel Composition

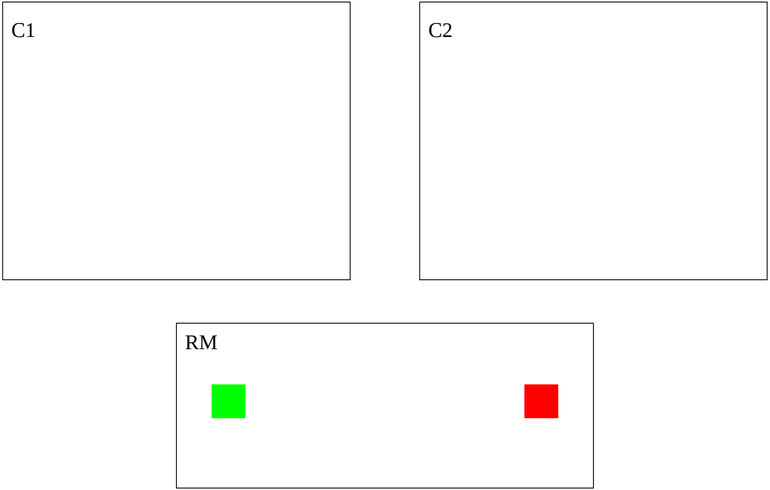
Example



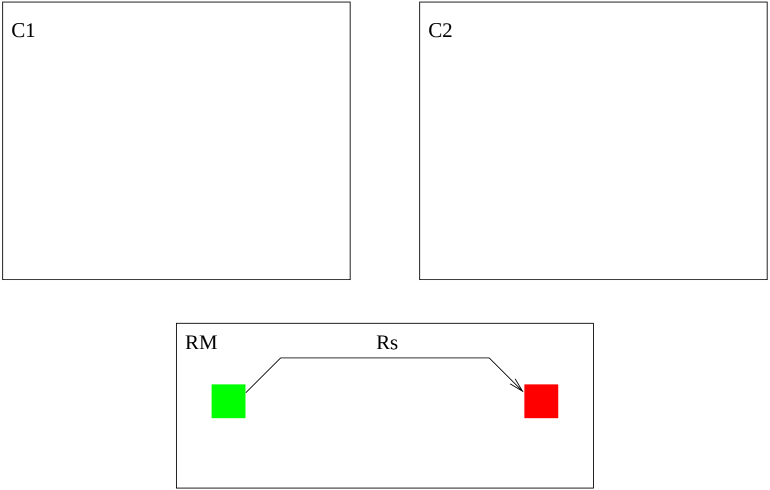
Example



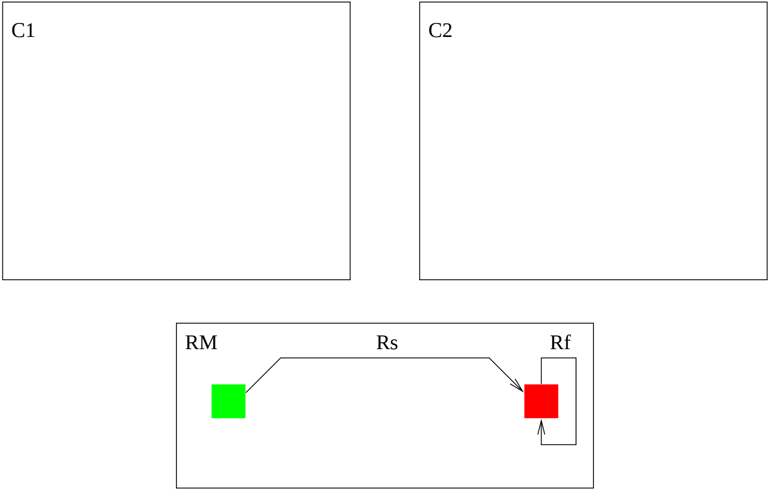
Example



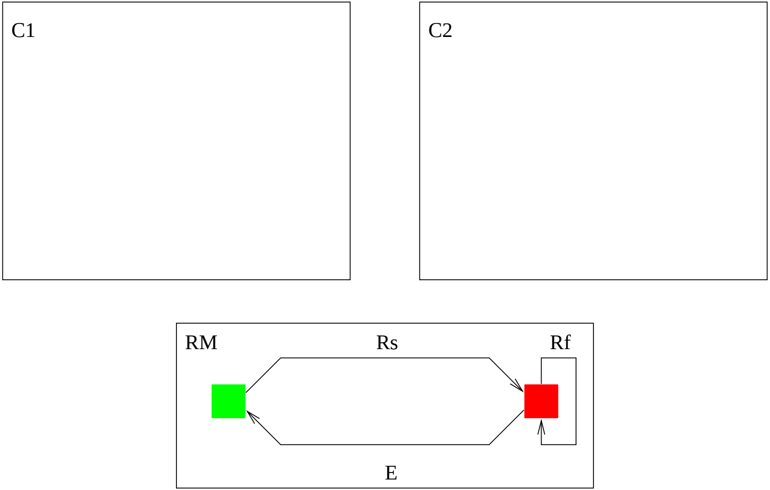
Example



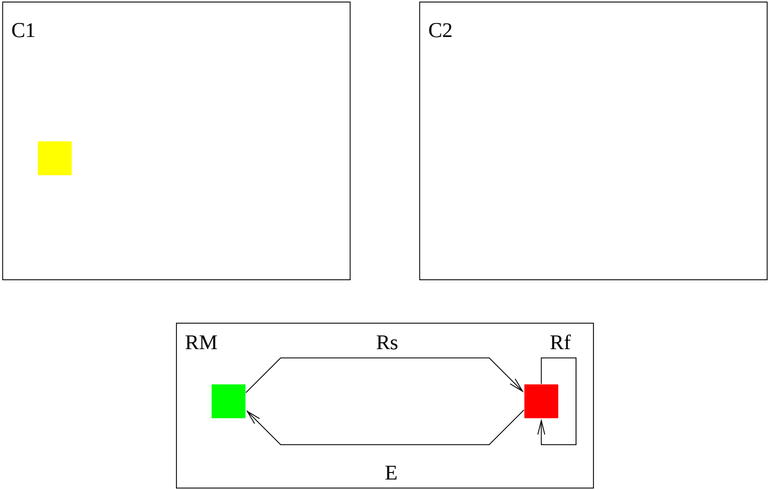
Example



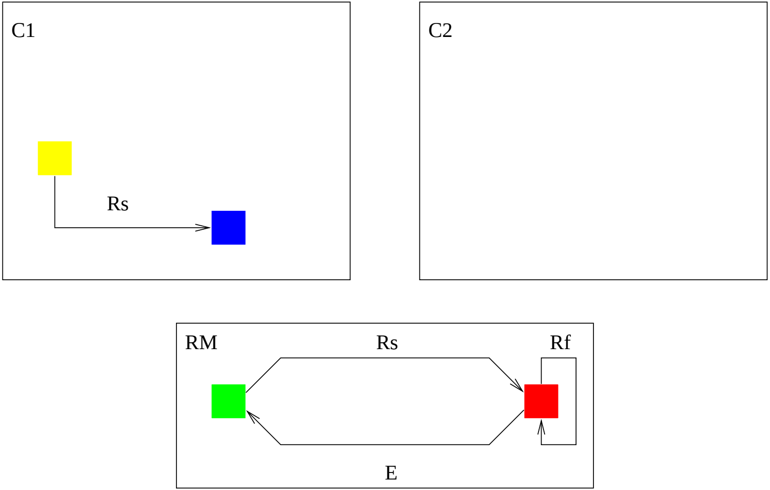
Example



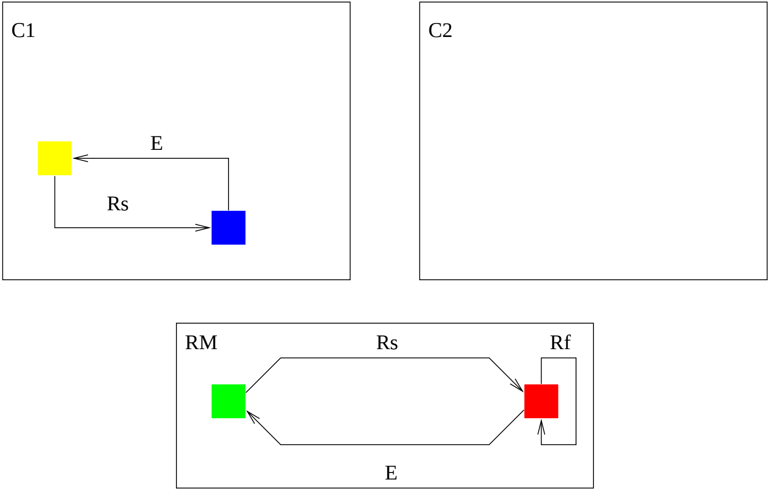
Example



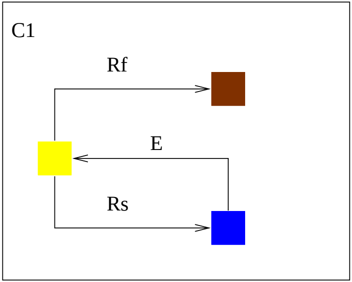
Example

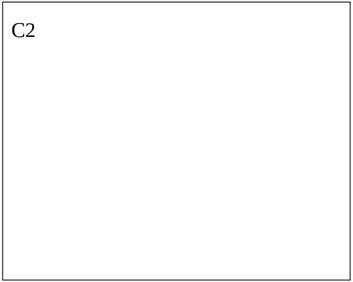


Example

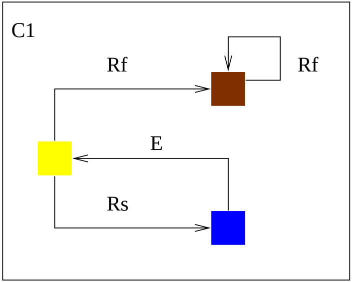


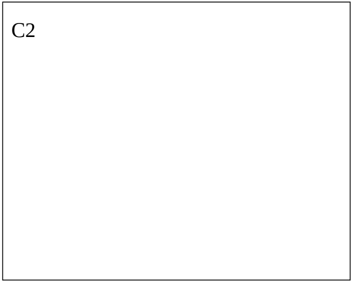
Example





Example





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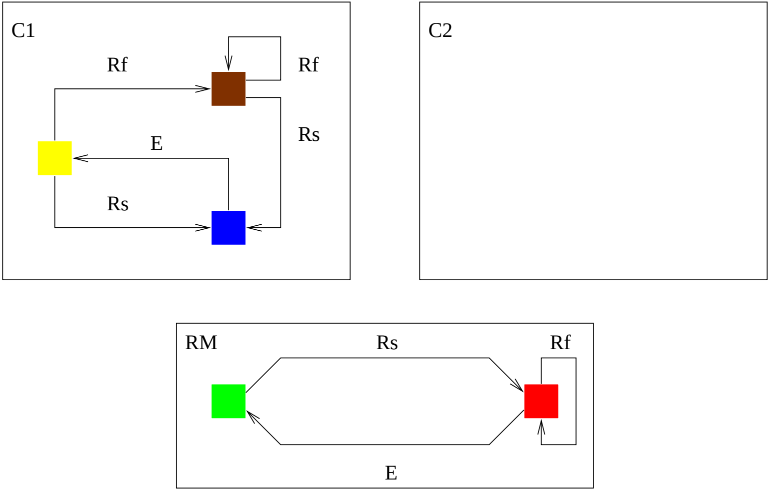

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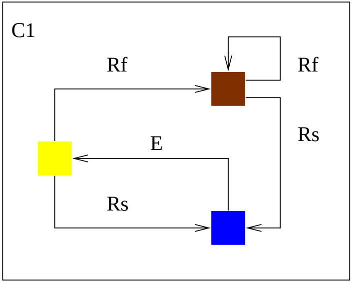
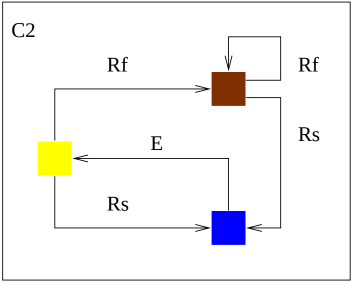
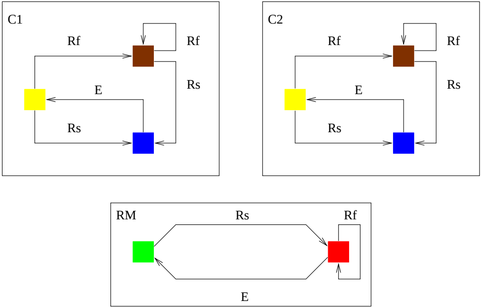




Example



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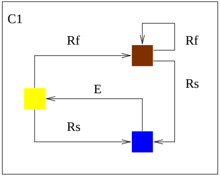
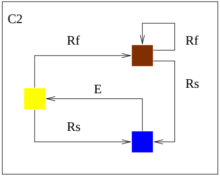
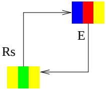
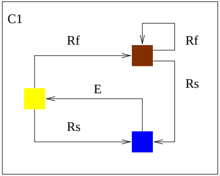
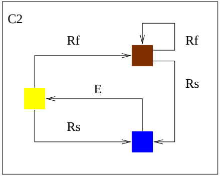


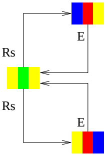




Example







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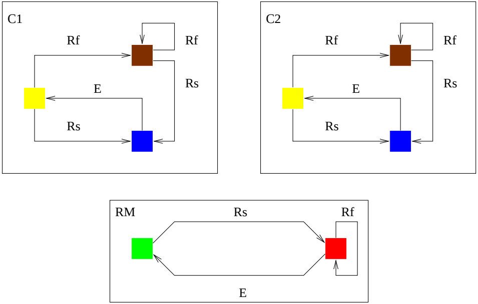

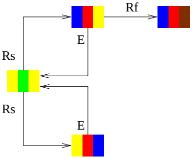

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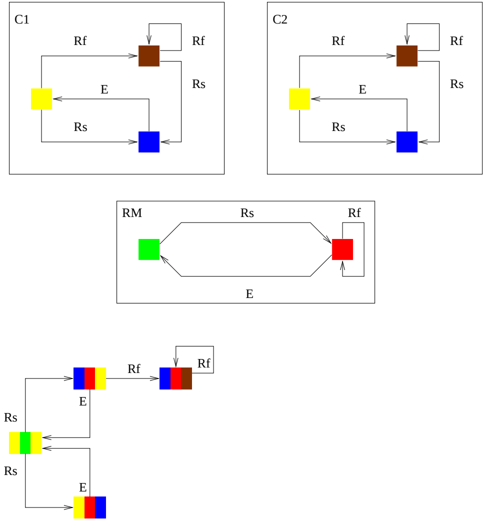

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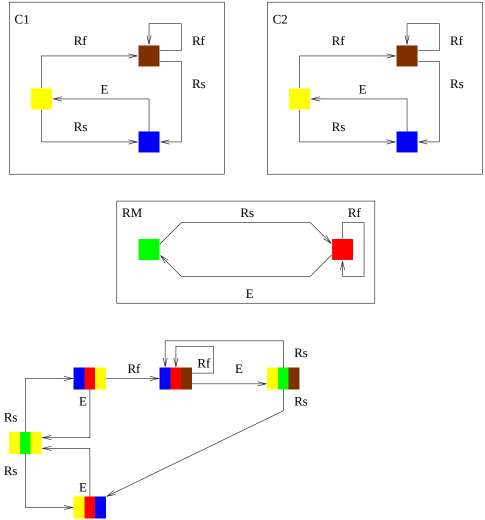

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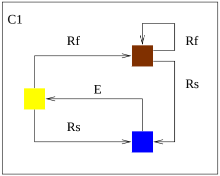
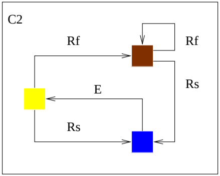
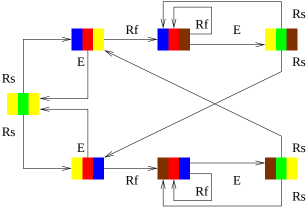
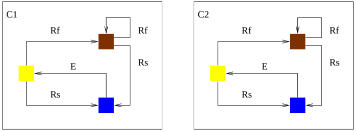

C1
Rf
Rf
Rs
E
Rs

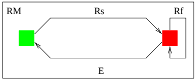

C2
Rf
Rf
Rs
E
Rs


RM
Rs
Rf
E

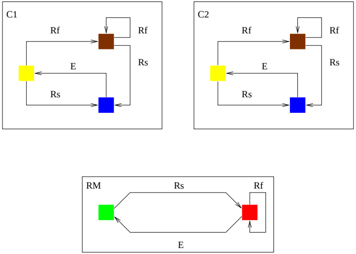

Rf
Rf
E
Rs
Rs
E
Rs
Rs
E
Rs
Rs
Rf
Rf
E
Rs

Example





Example

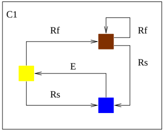


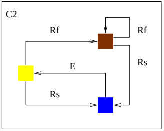
$\mathrm{Idle} \triangleq \mathrm{Rs}.\mathrm{Using} + \mathrm{Rf}.\mathrm{Retry}$

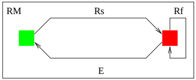
$\mathrm{Using} \triangleq \mathrm{E}.\mathrm{Idle}$

$\mathrm{Retry} \triangleq \mathrm{Rf}.\mathrm{Retry} + \mathrm{Rs}.\mathrm{Using}$

Example







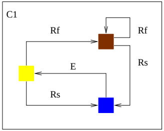
$\text{Idle} \triangleq \text{Rs}.\text{Using} + \text{Rf}.\text{Retry}$

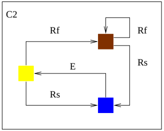
$\text{Using} \triangleq \text{E}.\text{Idle}$

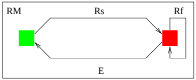
$\text{Retry} \triangleq \text{Rf}.\text{Retry} + \text{Rs}.\text{Using}$

$\text{C1} \triangleq \text{Idle}$

Example







$\text{Idle} \triangleq \text{Rs}.\text{Using} + \text{Rf}.\text{Retry}$

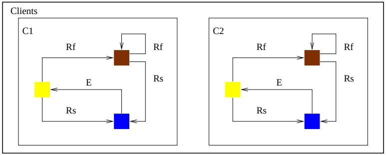
$\text{Using} \triangleq \text{E}.\text{Idle}$

$\text{Retry} \triangleq \text{Rf}.\text{Retry} + \text{Rs}.\text{Using}$

$\text{C1} \triangleq \text{Idle}$

$\text{C2} \triangleq \text{Idle}$

Example



$\text{Idle} \triangleq \text{Rs}.\text{Using} + \text{Rf}.\text{Retry}$

$\text{Using} \triangleq \text{E}.\text{Idle}$

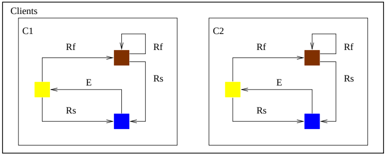
$\text{Retry} \triangleq \text{Rf}.\text{Retry} + \text{Rs}.\text{Using}$

$\text{C1} \triangleq \text{Idle}$

$\text{C2} \triangleq \text{Idle}$

$\text{Clients} \triangleq (\text{C1} \mid [\,] \mid \text{C2})$

Example



$\text{Idle} \triangleq \text{Rs}.\text{Using} + \text{Rf}.\text{Retry}$

$\text{Using} \triangleq \text{E}.\text{Idle}$

$\text{Retry} \triangleq \text{Rf}.\text{Retry} + \text{Rs}.\text{Using}$

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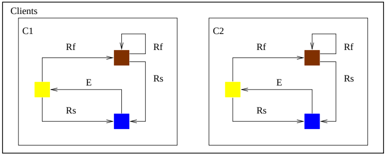
$\text{C2} \triangleq \text{Idle}$

$\text{Clients} \triangleq (\text{C1} \mid [\,] \mid \text{C2})$

$\text{Free} \triangleq \text{Rs}.\text{InUse}$

$\text{InUse} \triangleq \text{Rf}.\ \text{InUse} + \text{E}.\text{Free}$

Example



$\text{Idle} \triangleq \text{Rs}.\text{Using} + \text{Rf}.\text{Retry}$

$\text{Using} \triangleq \text{E}.\text{Idle}$

$\text{Retry} \triangleq \text{Rf}.\text{Retry} + \text{Rs}.\text{Using}$

$\text{C1} \triangleq \text{Idle}$

$\text{C2} \triangleq \text{Idle}$

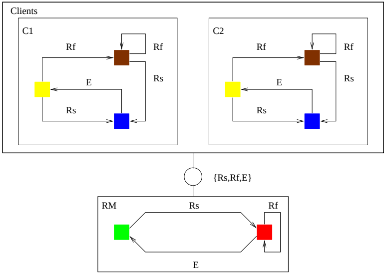
$\text{Clients} \triangleq (\text{C1} \mid [\,] \mid \text{C2})$

$\text{Free} \triangleq \text{Rs}.\text{InUse}$

$\text{InUse} \triangleq \text{Rf}.\ \text{InUse} + \text{E}.\text{Free}$

$\text{RM} \triangleq \text{Free}$

Example



$\text{Idle} \triangleq \text{Rs.Using} + \text{Rf.Retry}$

$\text{Using} \triangleq \text{E.Idle}$

$\text{Retry} \triangleq \text{Rf.Retry} + \text{Rs.Using}$

$\text{C1} \triangleq \text{Idle}$

$\text{C2} \triangleq \text{Idle}$

$\text{Clients} \triangleq (\text{C1} \mid [\,] \mid \text{C2})$

$\text{Free} \triangleq \text{Rs.InUse}$

$\text{InUse} \triangleq \text{Rf. InUse} + \text{E.Free}$

$\text{RM} \triangleq \text{Free}$

$\text{System} \triangleq \text{RM} \mid [\text{Rs,Rf,E}] \mid \text{Clients}$

A *process algebraic* approach to *system modelling*

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1. Algebraic terms

A *process algebraic* approach to *system modelling*

1. Algebraic terms, defined via a Formal syntax

A *process algebraic* approach to *system modelling*

1. Algebraic terms, defined via a Formal syntax, often with **graphical tool support**;

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2. LTS, the reference Mathematical Objects

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A *process algebraic* approach to *system modelling*

1. Algebraic terms, defined via a Formal syntax, often with **graphical tool support**;
2. LTS, the reference Mathematical Objects, equipped with Behavioural Relations, i.e. Formal Preorders, Equivalences, and Congruences
3. A mapping of terms to LTS

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1. $\mathcal{P}$: the set of states defined by the above grammar
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For $S \in \mathcal{P}$, let $\mathcal{R}_S$ be the set of states in $\mathcal{P}$ which are reachable from S via $\longrightarrow$, $LTS_S \stackrel{\text{def}}{=} (\mathcal{R}_S, A_t, \longrightarrow \cap (\mathcal{R}_S \times A_t \times \mathcal{R}_S), S)$

Advantages

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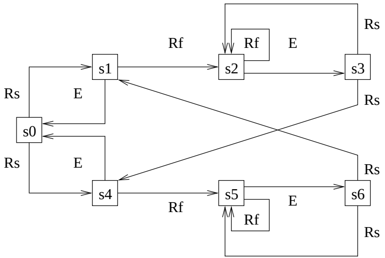
- Axiomatic reasoning

A
Temporal Logics
Approach to
Requirement Specification

Computation Tree Structures

- Description of behaviour of a system by means of the set of its computations:
  - Computation: (possibly) infinite sequence of states which are reached, and transitions which take place during a single system run from the initial state;
  - Set of computations: represented as an (infinite) tree;
    - A *Computation Tree* associated to each system
    - A computation of the system: a *path* in the CT

Computation Tree Structures



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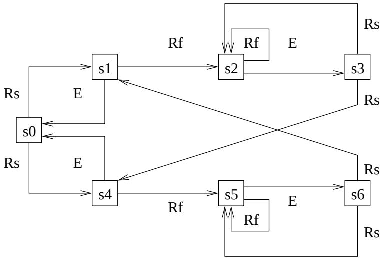

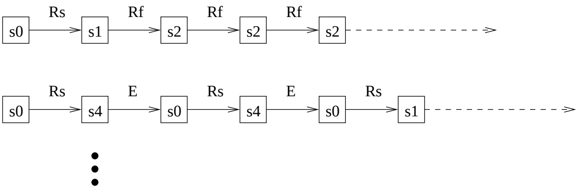

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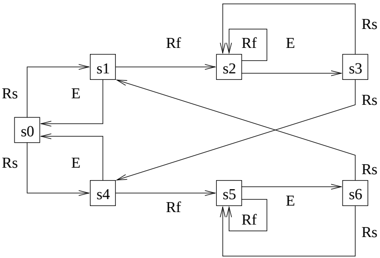
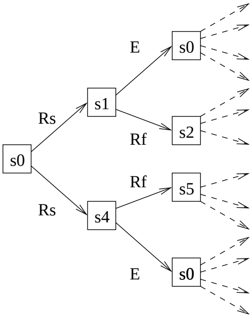
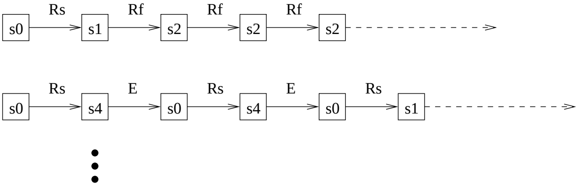
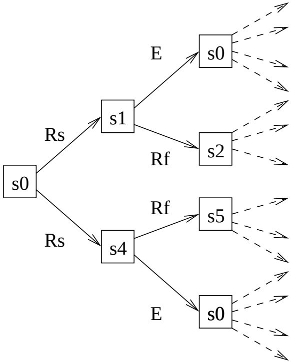






Computation Tree Structures

- **Graphical notation** (...)



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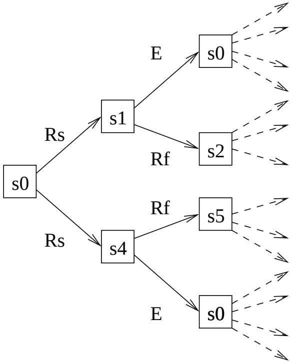

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Computation Tree Structures

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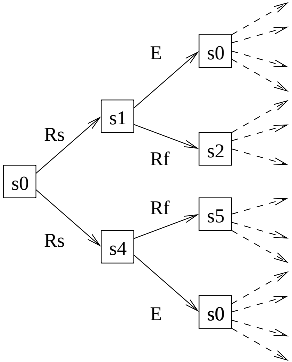

- **Mathematical definition**

  e.g. in the framework of formal (ω-)languages or Computation Trees

Properties of paths

REMEMBER: paths represent system computations (traces, logs ...)!

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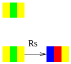

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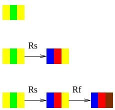

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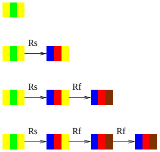

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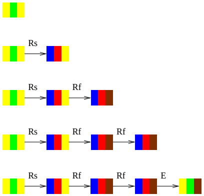

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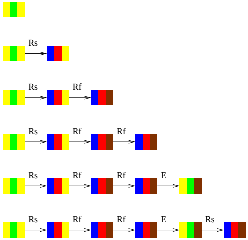

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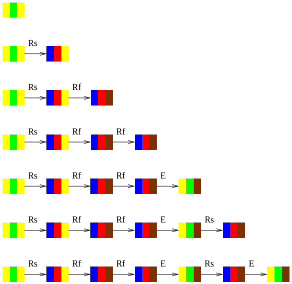

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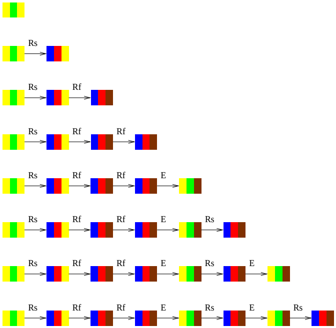

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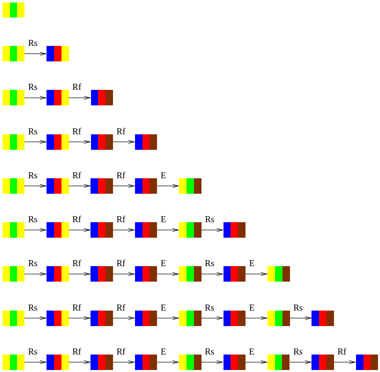

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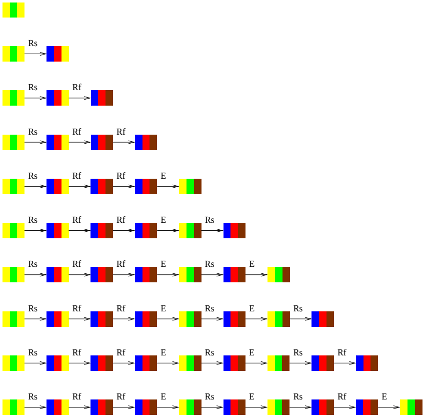

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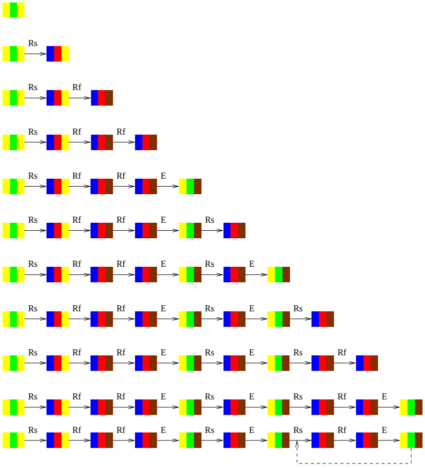

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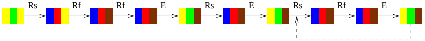

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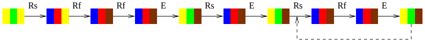

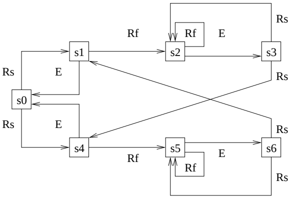

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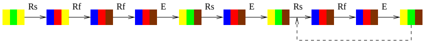

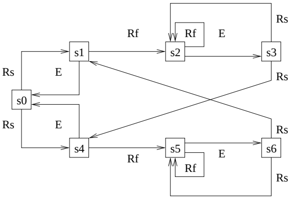

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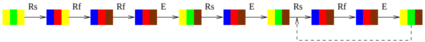

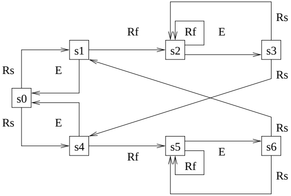

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that is, in ω-languages notation

s0.Rs.s1.(Rf.s2)^2.E.s3.Rs.s2.E.s3(Rs.s2.Rf.s2.E.s3)$^\omega$

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From logic formulae to computations via Formal Semantics

$$(\underline{in}(s0) \vee \underline{in}(s1) \vee \underline{in}(s2)) \;\; \mathcal{U} \;\; \underline{in}(s3)$$

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(Grammar)

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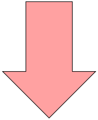

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Formal Semantics definition
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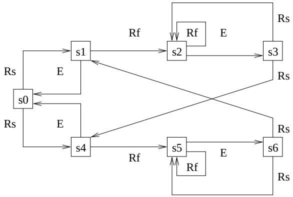

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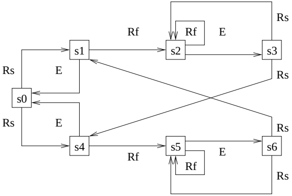

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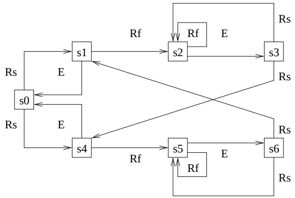

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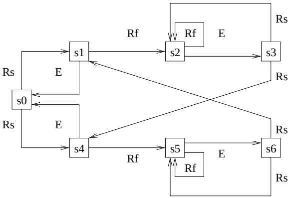

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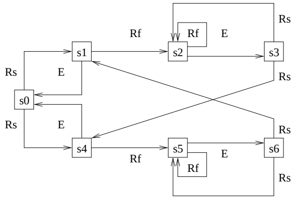

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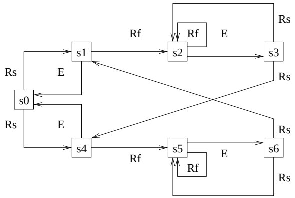

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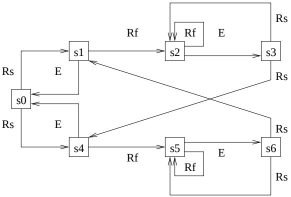

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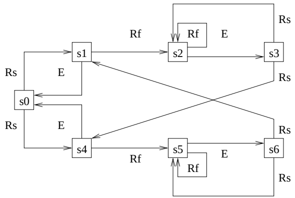

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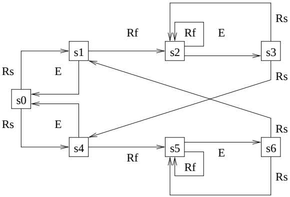

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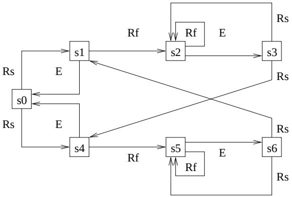

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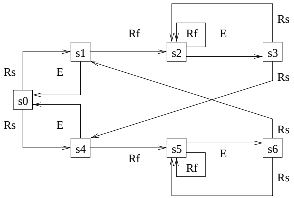

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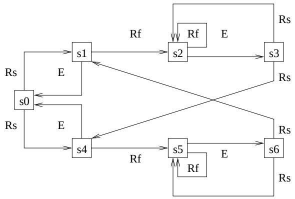

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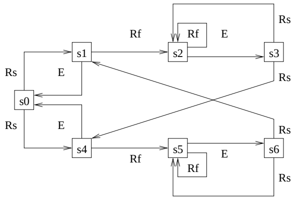

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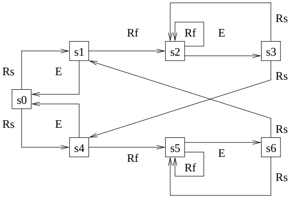

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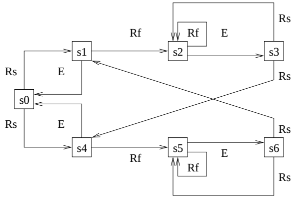

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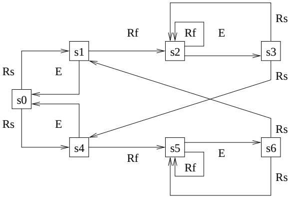

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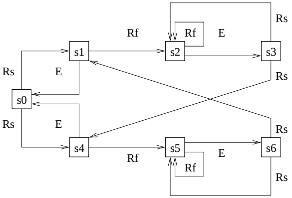

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A *Temporal Logics* approach to *Requirements specification*

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Formulae manipulation: $\forall\Box\Phi \Rightarrow \exists\Box\Phi$

Formal Methods Model Checkers

Computer support: mechanization of formal manipulation

Automatic Model Checking

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Requirements Specification
(technical specification)

Automatic Model Checking

Requirements Specification
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Design Specification
(system model)

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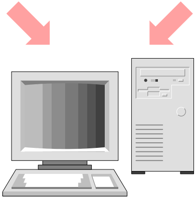

Automatic Model Checking

Requirements Specification
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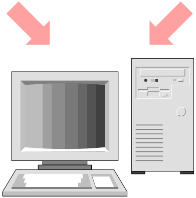

OK!

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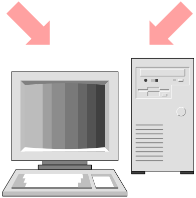

ERROR!!

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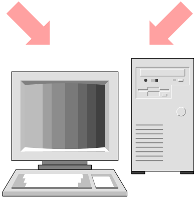
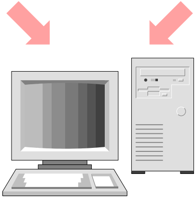


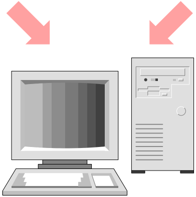
ERROR!!
+
counter example!!

Automatic Model Checking



Automatic Model Checking

System $\triangleq$
RM | [Rs,Rf,E] | Clients



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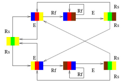

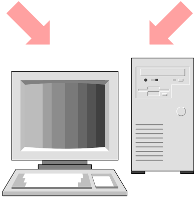

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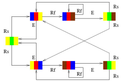

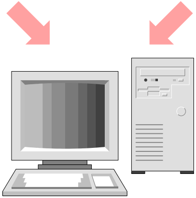

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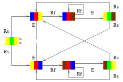

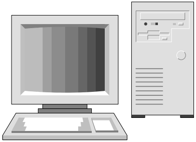

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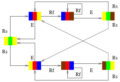

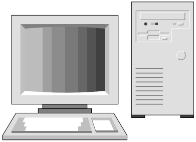

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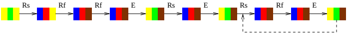

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Given: $\mathcal{K} = (S, A_s, L, \rightarrow)$ and Φ, returns: $Sat(\Phi) \stackrel{\text{def}}{=} \{s \in S \mid s \models \Phi\}$.

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Classical FM have been successfully used for modeling and analyzing functional aspects of complex systems, for example:

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Bill Gates, April 18, 2002.
Keynote address at WinHEC 2002

Timed/Probabilistic/Stochastic Extensions of Process Algebraic System Modelling and Temporal Logic Requirement Specification

Extensions of FM

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From algebraic terms to LTS via Formal Semantics.

Formal Syntax definition
(Grammar)

$$
\begin{aligned}
s0 &\triangleq a.s1 + d.s2 + a.s4\\
s1 &\triangleq b.s0\\
s2 &\triangleq b.s3\\
s3 &\triangleq a.s0 + a.s5\\
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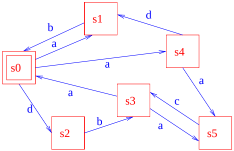
s5 &\triangleq c.s3
\end{aligned}
$$

Formal Semantics definition
(Logic deduction system)

Mathematical Objects
(LTS)



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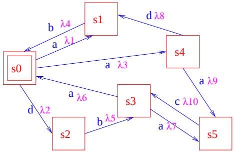

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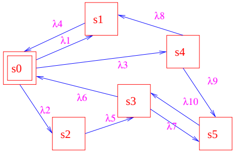

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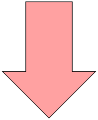

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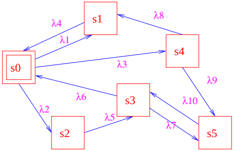

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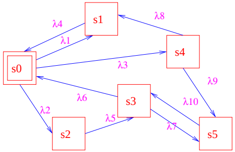

A *process algebraic* approach to *stochastic* system modelling

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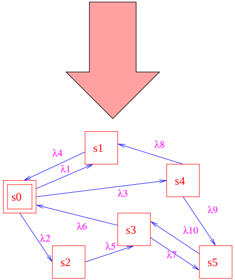
From logic formulae to CTMC via Formal Semantics.

Formal Syntax definition
(Grammar)

Formal Semantics definition
(Logic deduction system)

Mathematical Objects
(CTMC, Cones, Cilynders)

$$s1 \models \exists((\underline{in}(s1) \vee \underline{in}(s2) \vee \underline{in}(s3)) \;\; \mathcal{U} \;\; \underline{in}(s4))$$



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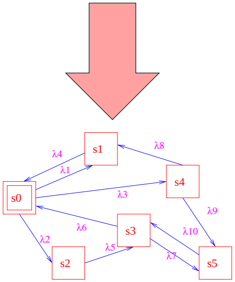

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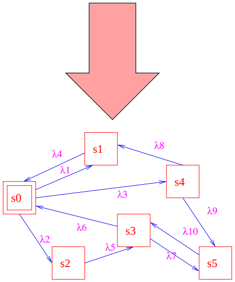

A *stochastic logics* approach to *non-functional* Requirement specification

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$$\Phi ::= \mathcal{A} \;\Big|\; \neg\Phi \;\Big|\; \Phi \wedge \Phi \;\Big|\; \Phi \vee \Phi \;\Big|\; \mathcal{S}_{\bowtie p}(\Phi) \;\Big|\; \mathcal{P}_{\bowtie p}(\varphi)$$
$$\varphi ::= \mathbf{X}^t\, \Phi \;\Big|\; \Phi \;\; \mathcal{U}^t\, \Phi$$

2. CTMC, the reference Mathematical Objects, equipped with proper theory

3. A relation between formulae and CTMC, the Formal Semantics definition

$$\gamma \models \mathbf{X}^t\, \Phi \text{ iff } \gamma[1] \text{ is reached by time } t \text{ and } \gamma[1] \models \Phi.$$

A *stochastic logics* approach to *non-functional* Requirement specification

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$\gamma \models \Phi_1\;\;\mathcal{U}^t\,\Phi_2$ iff there exists $j \geq 0$ s.t. $\gamma[j]$ is is reached by time t, $\gamma[j] \models \Phi_2$, and $\gamma[i] \models \Phi_2$, for all $0 \leq i < j$

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$$s \models \mathcal{P}_{\geq p}(\varphi) \text{ iff } \mathbb{P}\{\gamma \mid \gamma \models \varphi\} \geq p$$

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$$s \models \mathcal{P}_{<p}(\varphi) \text{ iff } \mathbb{P}\{\gamma \mid \gamma \models \varphi\} < p$$

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   $s \models \mathcal{S}_{\geq p}(\Phi)$ iff the probability to be in a state s' s.t. $s' \models \Phi$, in the *long run* starting from s, is $\geq p$.

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1. Logic formulae, defined via a Formal syntax, e.g. Baier et al. CSL-like:

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4. **Automatic verification**

A *stochastic logics* approach to *non-functional* Requirement specification

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2. CTMC, the reference Mathematical Objects, equipped with proper theory
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4. **Automatic verification, i.e. Stochastic Model Checking**

THANK YOU!!

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